	W1	W2	W3	
F1	2	2 / 7	3 / 4	5/2/0
f2	3	3	8 / 1	8/0
f3	5	7 / 4	7	7/0
f4	7 / 1	6	7 / 2	14/7/0
	7/0	9/2/0	18/10/3/0	

Total transportation Cost

$= 2\times7 + 3\times4 + 8\times1 + 7\times4 + 7\times1 + 7\times2$

$= 14+12+8+28+21+7$

$\Rightarrow 28 + 20 + 28 + 7$

$\Rightarrow 48 + 28 + 7 = 76 + 7 \Rightarrow 83$

Method III

Vogels Approximation Method 6-

Ques

	W1	W2	W3	Available	Penalties			
f1	3 / 2	2 / 7	4	5/2/0,	(2)	2	2	(5)
f2	3	3	8 / 1	8/0	(2)	—	—	—
f3	5	7 / 4	7	7/0	(1)	1	1	1
f4	4 / 1	6	10 / 2	14/4/0	(1)	1	(5)	—
Demand.	7/3/0	9/2/0	18/10/0					
Penalties	(1)	(1)	(1)					
	1	1	(2)					
	1	2						
	3	3						

choose two minimum value, in each row, and each column and write differ b/w them. and select the maximum difference. and allocate the cell.

Now repeat the process again,

Total Transportation Cost $\Rightarrow 3\times2 + 2\times7 + 8\times1 + 7\times4 + 4\times1 + 10\times2$

$= 6 + 14 + 8 + 28 + 4 + 20$

$\Rightarrow 20 + 36 + 24$

$= 44 + 36 = 80$ (which is minimum as com two other Methods).

Ques 10 Suggest optimum solution to the following assignment problem and also the maximum sales.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline A & 44 & 80 & 52 & 60 \\ B & 60 & 56 & 40 & 72 \\ C & 36 & 60 & 48 & 48 \\ D & 52 & 76 & 36 & 40 \end{array}$$

Solⁿ. Subtract the each element of the given matrix from the greatest element 80 we get

Step 1

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline A & 36 & 0 & 28 & 20 \\ B & 20 & 24 & 40 & 8 \\ C & 44 & 20 & 32 & 42 \\ D & 28 & 4 & 44 & 40 \end{array}$$

Step 2

Now subtract the minimum value corresponding to each row, we get

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline A & 36 & 0 & 28 & 20 \\ B & 12 & 16 & 32 & 0 \\ C & 24 & 0 & 12 & 22 \\ D & 24 & 0 & 40 & 36 \end{array}$$

Step 3

Now subtract the minimum value corresponding to each column we get.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline A & 24 & \cancel{0} & 16 & 20 \\ B & \boxed{0} & 16 & 20 & \cancel{0} \\ C & 12 & \cancel{0} & \boxed{0} & 22 \\ D & 12 & \boxed{0} & 18 & 36 \end{array}$$

Step 4: Giving zero assignment in the usual manner, we get the following matrix →

Step 5: draw minimum no of lines (horizonal and vertical) to cover all the zeros at least once.

Step 6: Since the smallest element among all uncovered elements is 6, so subtract 12 from all uncovered elements and adding it every element that lies at intersection of two lines and leave remaining. we get

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline A & 12 & \boxed{0} & 04 & 148 \\ B & 0 & 28 & 20 & \boxed{0} \\ C & 12 & 12 & \boxed{0} & 22 \\ D & \boxed{0} & 0 & 18 & 30 \end{array}$$

A → 2
B → 4
C → 3
D → 1

$80 + 72 + 48 + 52$

$\Rightarrow 152 + 100$

$\Rightarrow 252$ Rs (Maximum sale)

Theorems.

Existence of feasible solutions:-

A necessary and sufficient condition for the existence of feasible solution of a $m \times n$ transportation problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Proof:

Necessary Condition:- let there exist a feasible solution of the transportation problem than,

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2 \ldots m$$

and

$$\sum_{i=1}^{m} x_{ij} = b_j \quad i = 1, 2 \ldots n.$$

Summing over all i and j respectively, we get

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i \quad \text{and} \quad \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} b_j.$$

but $\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij}$

Hence, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$

Sufficient Condition:- If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

then there exist a feasible solution of the transportation problem.

let $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = K$ (say)

If $x_{ij} = \lambda_i b_j$ for all $i = 1, 2 \ldots m$ and $j = 1, 2 \ldots n$ where $\lambda_i \neq 0$ is any real number then,

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \lambda_i b_j = \lambda_i \sum_{j=1}^{n} b_j = K\lambda_i \Rightarrow \lambda_i = \frac{1}{K} \sum_{j=1}^{n} x_{ij} = \frac{a_i}{K}.$$

Thus $x_{ij} = \lambda_i b_j = \frac{a_i b_j}{K} \geq 0$ for all i and j. $a_i > 0$, $b_j > 0$ and $K > 0$.

Hence, a feasible solution of the transportation problem exist.

Out of $(m+n)$ equations, in $m \times n$ transportation equation, one is redundant and remaining $m+n-1$ form a linearly independent set.

Proof: Consider the following $m+n-1$ equation of a $m \times n$ transportation problem.

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2 \ldots m \qquad m \text{ row equation} \quad \text{—(1)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2 \ldots (n-1) \qquad (n-1) \text{ row equation} \quad \text{—(2)}$$

where, $\sum_{j=1}^{n} b_j = \sum_{i=1}^{m} a_i$ (3) By these $m+n-1$ equation given in (1) and (2) and eq^n (3) we shall get the remaining n^{th} column equation, Adding m-row equation given in (1) we get.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i \quad \text{—(4)}$$

and

$$\sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n-1} b_j \quad \text{—(5)}$$

Subtract 5 from 4,

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n-1} b_j$$

$$\sum_{i=1}^{m} \left[\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n-1} x_{ij} \right] = \sum_{j=1}^{n} b_j - \sum_{j=1}^{n-1} b_j \quad \text{using 3.}$$

$$\sum_{i=1}^{m} \left\{ \cancel{\sum_{j=1}^{n-1} x_{ij}} + x_{in} - \cancel{\sum_{j=1}^{n-1} x_{ij}} \right] = \cancel{\sum_{j=1}^{n-1} b_j} + b_n - \cancel{\sum_{j=1}^{n-1} b_j}$$

$$\sum_{i=1}^{m} x_{in} = b_n$$

i.e. if $m+n-1$ satisfied them $m+n$ eq^n also satisfied,